The Value of Pi in Rabbinical Mathematics

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Abstract

There are notable discrepancies between the estimation of the value of Pi in traditional Judaic sources and the common knowledge of the eras in which they were written. This paper attempts to understand the philosophical principles behind the Rabbinical mathematics that led to these estimations and build an encompassing theory of the attitude to mathematics within Judaism. Several textual sources are analysed, before two diametrically opposite approaches are suggested. Two approaches to mathematics in Rabbinic Judaism – the 'Scientific' and 'Religious' – are defined and posited, each assessed for its merits and pitfalls whilst attempting to place scholars as proponents of either approach. Ultimately, the authors conclude with a philosophy that utilises both approaches and resolves many of the issues raised.

Introduction

The most famous mathematical constant has been a source of fascination and a challenge for mathematicians throughout the centuries. For Judaic thought, too, this has been a wonder, albeit in a slightly different manner. This paper will attempt to formulate the principles and philosophies driving the rabbinic interaction with the approximation of Pi in particular, as a paradigm for an approach to exactness, mathematics, and the sciences in general.

Jewish practice is no stranger to measurement and precision. The minutiae of day-to-day law – of which there are 613 basic commandments and a finitely large number of extensions – have been hotly debated, thoroughly investigated, and painstakingly measured for thousands of years. Each generation additionally finds itself presented with its own novel considerations and challenges. These discussions pervade every discipline, whether it be philosophy, psychology, astronomy, anatomy, or engineering. It should therefore come as a surprise that when dealing with the matter of the ratio of a circle's diameter to its circumference, the Bible and Talmudic sages seem all too content to halt at a very coarse estimate.

This paper opens with the background of Judaic estimates of the value of Pi, beginning with the only source explicit in the Bible, before analysing further two Talmudic discussions. Each text will be analysed briefly and contrasted with the common knowledge of Pi during that era. Attempts to resolve the inaccuracies in approximation will be presented, after which the authors suggest two potential philosophies that would drive this discrepancy. Each approach will be developed and supported by rabbinic literature, eventually being considered thoroughly before a conclusion is reached.

King Solomon's Copper Sea

The Bible provides the first Judaic source for the approximation of Pi:

> *"And he made the sea of cast [metal], ten cubits from edge to edge, circular all around; and five in cubits its height, and a line of thirty cubits would encircle it all*

around…And its thickness a cubit, like the work of the edge of the cup of a rose flower – two-thousand baths it could contain." (Kings I, 7:23,26)[1]

The "sea" of which the Bible speaks is a large vessel designed to contain water for the ritual washing of priest's hands before and during the Temple service. The ratio between the circumference of the "sea" and its diameter is given as 3:1[2]. Considering that already in 1650 B.C.E – approximately 700 years before King Solomon's temple – the Egyptians, a nation with whom King Solomon had close ties, had approximated Pi to $\frac{256}{81}$ (Van der Waerden, (1976), and that this multinational building project gathered the most talented engineers in the country, it is difficult to posit that this was the most accurate approximation that King Solomon could conjure. Therefore, two approaches are presented in attempt to resolve this inaccuracy:

i. **The Bible was entirely accurate in its descriptions of the world**

This is a difficult approach since it is well known (both now and then) that the ratio of a circle's circumference to its diameter is not exactly 3. If we were to accept that the ratio given is exact, we would be forced to conclude that the "sea" was not circular, but rather elliptical in shape. This, however, contradicts the verse that the "sea" was circular and thus this approach meets an impasse whereby either the description of the ratio is inaccurate, or – no better – the geometric description is incorrect which renders us having replaced one serious issue with another.

ii. **The Bible is not attempting to calculate the ratio of a circle's circumference to diameter. The Bible is not a mathematical work and was not written to solve mathematical problems. It is highly likely that the temple engineers were aware of, and used, a more accurate measurement of Pi, but when reporting the details, the Book of Kings gives a rounded value for either the diameter, the circumference, or both.**

This approach falls in line with the often-quoted Talmudic principle that the Bible speaks in human terms, in a manner that is comprehensible to the average person[3] (Bava Metzia, 31b)[4]. It also avoids the aforementioned impasse by not even laying the claim to accuracy in measurements.

Several other approaches have been adopted. Amongst them is Engleson (2017), who engages in several attempts to explain this discrepancy. He begins by trying to minimise the error: conversions of different biblical measurements lead to the conclusion that volume of the copper sea was 450 cubic cubits, with the lower 3 cubits being square in shape and only the upper 2 circular.[5] Thus, by taking Pi at its known value today, we receive a volume of approximately 457.07 cubic cubits[6], which

[1] For instances where the source text language is Hebrew or Aramaic, we have translated the text ourselves. For biblical references, we have taken the Hebrew text from the Jewish Publication Society (1999) edition as appears in the bibliography. We have preferred, however, our own translation, for various contextual and accuracy-related reasons.

[2] From here forward, we refer to all ratios of the form x: y simply as x when the value of y is 1.

[3] This is a fascinating topic that deserves more attention but is beyond the scope of this paper.

[4] When citing Talmudic texts, we have adopted the format of (Tractate Name, Folio number). For tractate names, we have adopted the Hebrew title spelled out in English. Translating the name is convoluted and often meaningless. All references are to the Babylonian Talmud unless otherwise stated. All references to the Talmud are to Steinsaltz, A., et al. (2012) who edited the edition published by Shefa Foundation as stated in the bibliography.

[5] For the lower half: $volume_{sea} = 3 \cdot 10 \cdot 10 = 300\ cubits^3$. The upper half is a cylinder and (with an approximation of 3 for π) gives $volume_{sea} = 3 \cdot 5^2 \cdot 2 = 150$ for a total of 450 cubits.

[6] Replacing 3 in the latter equation in footnote (5) with the true value of Pi provides this approximation.

is not a bad estimation by any means. He continues in this manner, taking different biblical readings to further minimise the approximation error, indicating that the biblical estimation was not far off at all.

Engleson (2017) then proceeds to engage in mysticism – using a ancient system known as *Gematria* in which each Hebrew letter has a numerical value, he shows that the ratio between the value of the words in the text, and the value of the way it is traditionally read[7] multiplied by the Pi approximation given in the text, is an exceptional estimate. We have:

$$\frac{written\ value}{reading\ value} \cdot given\ ratio = \frac{111}{106} \cdot 3 = 3.1415$$

This is, however, ultimately rejected.

The authors, in unanimity, prefer resolution (ii) presented above, certainly over resolution (i), owing to the impasse it presents, and over Engleson's (2017)[8] suggestions for its simplicity and directness – an instance of Occam's Razor perhaps. There is no need to engage in biblical apologetics and force readings that are difficult to see in the original text when a perfectly plausible explanation exists – especially one that's consistently espoused across other Rabbinic Literature.

It is important to note, however, that the above biblical verses are hardly an attempt at an approximation of Pi. In line with the authors' adopted approach resolving the inaccuracy in the ratio – that is that the Bible merely provides a description and lays no claim to mathematical accuracy – the focus of the remainder of this paper will be dedicated to the Rabbinic approximation, which has further-reaching implications and is more likely an attempt at an accurate calculation.

A Crossbeam Above an Alleyway

The Mishna[9] describes the minimum length needed for a beam above the entrance to an alleyway to suffice considering the alleyway as outside of the public domain. It proceeds to discuss crossbeams that are not straight, and focuses in particular on a circular beam:

> *"Any [beam] that has in its circumference 3 handbreadths, has a width of one handbreadth." (Eiruvin, 1:5)*

We have here a rabbinic indication that the ratio of Pi is indeed given as 3. The Babylonian Talmud comments on the above:

> *"From where are these words derived? Rabbi Yoḥanan said that the verse said with regard to King Solomon: "And he made the sea of cast [metal], ten cubits from edge to edge, circular all around; and five in cubits its height, and a line of thirty cubits would encircle it all around" But is there not its edge? Rav Pappa said: With regard to its edge, it is written that the edge is as the petals of a rose-flower, "And its thickness a cubit, like the work of the edge of the cup of a rose flower – two-*

[7] There is an extra letter in the written text. Discussion of this discrepancy, and the manner in which the *Gematria* numeric system works, is well beyond the scope of this text.

[8] Engleson's ultimate resolution is not too different from the one we have adopted here. We are rejecting his proposed theories.

[9] The Mishna is the precursor to the Talmud, upon which it is based. We cite Mishnaic texts in the format of (Tractate Name, Chapter: Section). All references to the Mishna are to Kehati, & Tomaschoff (1995), editors of the edition published by the Department for Torah Education and Culture in the Diaspora, as stated in the bibliography.

thousand baths it could contain" But nevertheless, is there not a minimal amount [of the thickness of the edge]? When one calculates, from the inside are the calculations." (Eiruvin, 14a)

Thus, the Babylonian Talmud uses the biblical verse to prove this ratio as 3. At a first glance, this seems to contradict the claim above, that the Bible spoke in human language and was not attempting accuracy. In fact, the Talmud itself questions this ratio and goes on to mention that it does not consider the thickness of the "sea", which was given as a handbreadth[10] thick. The Talmud, however, seems content with the answer that the circumference was calculated from the inside of the "sea", thus negating the impact of the thickness of the walls.

It is important to note that the Mishna is a work that is concerned with accuracy and minutiae.[11] Unlike the Bible, it does not necessarily serve the goal of being accessible to all and, to that effect, was not written in a language that would be understandable to the average layman per se. This further heightens the question as to why the Talmud seems to be satisfied with a ratio of 3, but before an answer is offered, another Talmudic source is to be considered.

A Festival Booth

The Talmud (Sukkah, 7b-8a) discusses at length the minimum dimensions of a circular booth that would render it acceptable for use over the festival of *Sukkot*[12]. Although the plain text there[13] indicates once again that the ratio of a circle's circumference to its diameter is 3, Rabbi Yohanan provides a more interesting opinion, based on the number of people that can fit around the circumference of the booth. Understandings of this opinion vary, both in the Talmud itself and in various commentaries afterward. Although this approximation is certainly far better than 3 (Elishakoff & Pines, 2007), it seems clear from the text that the default reasoning of the Rabbis still takes the ratio at 3.

Although there are numerous further Talmudic examples of the ratio of Pi being used, the above will suffice the discussion at hand. It stands that the accepted Talmudic value for Pi is 3, which is somewhat surprising, given that Archimedes' inequalities of $\frac{233}{71} < \pi < \frac{22}{7}$ were ratified several centuries before the scholars of the Talmud. More surprisingly, there is also evidence of Mishnaic personalities stating this ratio as $\frac{22}{7}$ (Rabbi Nehemia, Mishnat HaMiddot)[14]. This leads to the presentation of two possible philosophies that drove the Rabbinic discussion regarding the value of Pi specifically, and their approach to mathematics in general. Although these approaches were generated from the philosophising and textual analysis of the authors, they are not too dissimilar to the conflicting approaches in the schism of Judaic thought proposed by Slifkin (2021).

[10] Approximately 7-8cm

[11] For example, a ritual bath measuring 40 *seah* (a measure of volume) is valid, yet even a small amount less is not.

[12] Sukkot, lit. booths, is a Jewish festival celebrated for a week shortly following the Jewish New Year. It is characterised by building small huts outside in which eating and sleeping takes place over the festival duration.

[13] We did not quote the text as a result of its length and back-and-forth nature.

[14] It is important to note that Mishnat Hamidot, A Mishnaic-era document discussing weights, sizes, volumes and measurements, was not authored by Rabbi Nechemia. The approximation mentioned here is attributed to him there. Reference to the source text in the bibliography is attributed to *Torat Emet Publishers*.

The Scientific Approach

The first of these philosophies is to be termed "the Scientific Approach". It holds that since the framework of Jewish ritual practice is defined by a specific set of laws and guidelines, accuracy is of the utmost importance. Wherever one can be exact, it is important to be so. (Note that the term 'Scientific' is used here very loosely.) This, of course, does not demand an adherence to the scientific method or modern principles of science and research, but is intended to capture the overall attempt at striving for specificity and exactness.

This attitude certainly integrates well into the broader philosophy of Jewish law seen in the Talmud, for example, the 13 principles of logical reasoning given by Rabbi Yishmael (Sifra: Techings of Rabbi Yishmael, 1:1). Jewish law is very specific in its measurements, timing and values – often with very little room for error (take, for example, any of the prohibitions of the Sabbath). This approach categorizes, as much as can be, in an orderly manner and is as specific as possible. It therefore must contend with the earlier question – how can it be that the Rabbinic estimation of Pi was wrong, and nonchalantly so? (It must be noted that there is no verbose attempt to reach a solution, rather to build a philosophical approach that prioritises accuracy and measurement.)

As mentioned above, Engleson (2017) is a proponent of such an approach. He posits several theories, all very meticulous, that range from slightly forced yet creative logical resolutions, all the way across to solutions more esoteric and mystical in nature. His primary focus though is resolving the biblical difficulty with King Solomon's copper "sea" and is less concerned with the Rabbinic discrepancies.

Maimonides too was a proponent of this methodology. Well-renowned for his knowledge of medicine, astronomy and Greek philosophy, Maimonides was a well-established rationalist that pedestaled the natural world and the various disciplines of science (not at the expense of religion, of course, but in a rich interweaving of the two). He writes in his Commentary on the Mishna (Eiruvin 1:5) that *'the exact relationship of the diameter to its circumference cannot be known and it is not possible to speak of it... its actual value cannot be perceived.*' The term "*its actual value cannot be perceived*" may resemble an early formulation of the definition of an irrational number.

Given the impossible nature of reaching an exact number, Maimonides satisfies himself with the value of $3\frac{1}{7}$ – a reasonably accurate estimation. Under this approach, there too would be room to argue that the Talmudic Rabbis were perhaps aware of the irrationality of Pi and resigned themselves to the simplicity of an approximation of 3, accepting that any other approximation would too be inaccurate.

This approach is characterised by the striving for exactness and accuracy as much as possible, and only where impossible, is there an allowance to compromise and settle for an approximation. The question collapses to a technical one of where is ideal to draw the line on a close enough approximation.

The implications of such a stance place great significance on the value of mathematical and scientific research within Judaism and Rabbinic studies. Developing mathematics is not only for the intellectual pursuit, but has theological significance, serving as a means to improve Jewish practice. Although it is true that the Jewish people have contributed immensely to the scientific disciplines, this phenomenon is less so from the Rabbinic world, and very rarely driven by theological ideals. The authors would perhaps hold back from calling the Rabbinic contribution to mathematics 'practically

nil' (S.B.[15], 1932), but would concur that Rabbinic scholars have, with a few notable exceptions, certainly limited their fields of influence to the confines of theology.

The Religious Approach

The second of these philosophies is to be termed "the Religious Approach". Noting of course, that the title of this approach was chosen for simplicity and has no intention of pitting science against religious values. Furthermore, the authors are not making the claim that proponents of the other philosophy were not religious individuals or inspired by science as a religious approach. Rather, that this naming convention helps to explains the differences between the two worldviews.

This approach gives priority to the theological. It is not concerned with the "true reality" of the world, nor is it interested in ratios of circumferences and diameters in practice. As far as this philosophy is concerned, since the Bible states that the ratio is 3, and this was verified by the Rabbis of the Talmud, this ratio must be regarded as exactly 3 whenever it needs applying. Proponents of this view would go as far as to say that this ratio is divinely ordained, or at least is an oral tradition from Moses himself and that whether or not this ratio is a reflection of reality is of little concern[16]. There is therefore very little mathematics to be discussed here, as this worldview is entirely devoid of the numerical that does not appear in the Talmud.

Within this philosophy, there is simply no contradiction between the Rabbinic approximation of Pi and others of their time periods and therefore there is nothing to question and no surprise to express. When the Talmud states that this ratio is 3, it was not at all referring to the mathematical value, but rather the one used for spiritual matters only. Although it is of no matter whether or not the Rabbis were aware of contemporary approximations of Pi, this makes it much easier to claim that they were and that the reason that they did not state them was because they were simply not relevant.

This approach, however, fails to address new developments within the sciences and their possibility to improve religious practice. For example, the Talmud (Yevamot, 92a) discusses the question of individuals of uncertain lineage – if there are more than one potential fathers, from do they inherit? (For example, if one of the potential fathers is a priest, are they subject to the laws of a priest?) The conclusion there is that the stringencies of each potential father are adopted as far as possible. Proponents of the "Religious Approach" would be forced to concede that per their philosophy, despite the possibilities offered in today's era of DNA testing – where not only can the situation be avoided entirely, but also that providing such clarity would improve the religious practice of the individual in question – the approach that must be taken is the one that mirrors the Talmudic conclusion. This of course seems counterintuitive – to de facto reject the "truth" as it were and endorse the stopgap approach borne out of the ignorance of the times.

[15] The reference is to a literature review written in the journal *Nature*. Only the author's initials are provided, their identity remains unknown. It is likely that they were a staff writer.

[16] There are, however, not a small number of Jewish authorities that attempt to reconcile this ratio with the natural world, arguing that the Bible would never err or provide an approximation, and it is rather hinting to mystical ideas. These resolutions go well beyond the topic at hand. Here we adopt a more moderate approach and limit this school of thought to the theological realm, both in thought and in practice.

This approach is only concerned with theology, and facts of the natural world are only relevant when they are critical and do not contradict previously stated religious texts. The implications of such a stance would be the utter disregard of mathematics and the sciences as a value, and most certainly if learning them comes at the expense of other "real" religious pursuits. This is not to say that the sciences do not have their place at all, of course; Rabbi S.Z. Auerbach famously took lessons in electrical engineering before writing his book of the laws of electricity and the Sabbath, but perhaps only so as to serve a direct theological outcome.

Conclusion

Of the two approaches offered; the "Scientific" and the "Religious", it is clear that whilst the former strives for accuracy, exactness and precision – and thus requires the natural world and the sciences to further its goals – the latter is indifferent to mathematics so long as it does not serve a purpose that aids religious practice or thought.

The former has advantages as it explains the broader philosophies of strict logic and precision within Rabbinic literature. It falls short however, on the question of the inaccuracy in estimating the value of Pi, as well as several other scientific errors noted in the Talmud. [17] It also fails to explain the distinct lack of Rabbinic literature in mathematics and the sciences in relation to the volume of the theological works; or is at the very least forced to concede that this view was not very prevalent throughout history.

The latter, on the other hand, easily explains away the question of Pi and all other scientific discrepancies, but remains slightly disconnected from reality, lacking the tools with which to engage in the world, and struggling to formulate a philosophy that allows for a serious consideration of the sciences, even at the cost of good religious practice.

The authors' approach in fact combines the two schools of thought, a hybrid per se. Indeed, the sciences are important, not only as an independent value, but too as a means for furthering religious practice. Exactness is important and mathematics enables such accuracy, therefore mathematics is too, a crucial component. Notably, however, the goal of both the Bible and the Rabbis is and always was to enable religious beliefs and practices. In so far as scientific precision furthers Judaism it is commendable, but when it becomes impractical for the average individual, it loses some of its relevance and a more palatable approximation suffices.

The Rabbinic disregard for the known approximations of Pi is now perhaps more understandable – when constructing a crossbeam or a festival booth, it is important to obtain a correct measurement, but only to a reasonable extent. It is rather harsh, and perhaps missing the forest for the trees, for the Rabbis to expect the layman to begin measuring $3\frac{1}{7}$ cubits and thus set the law at the rounded down 3. The only digression from the value of 3 we saw was Rabbi Yohanan with regard to festival booths, and although his value approximates Pi to much higher precision, he does not do so out of concern only for the value of Pi. He had found an ingenious method that allowed the booths diameter to be

[17] It is worthwhile to note, however, that several of the scientific misconceptions that the Rabbis held were commonly accepted scientific principles in the first few centuries of the common era, such as spontaneous generation and the geocentric model of Ptolemy.

calculated simply – the number of people that can sit around it – while at the same time, and perhaps inadvertently, improving the approximation of the ratio.

Finally, regarding the lack of Rabbinic output in the worlds of mathematics and the sciences, it should be noted that a significant increase has been seen through the recent decades. The authors hope that this mutual relationship continues to grow for long into the future.

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